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Devonian.				
Catskill (IX)			1300′	8717′
Chemung		(VIII)	7342′	
Portage				
Hamilton				
Upper Helderburg				
(Corniferous)				
Oriskany Sandstone (VII)			75′	
Upper Silurian.				
Lower Helderburg Limestone (VI)			1050′	2555′
Salina		(V)	730′	
Niagara?				
Clinton				
Medina Sandstone	275′	(IV)	775′	
Medina shales *	500′	(in part)		
Total height of section				15,201′

Photodynamic Notes, III. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

(*Read before the American Philosophical Society, June 17, 1881.*)

46. *Stellar Relations of the Corona Line and of Mass.*

The laws of harmonic æthereal oscillation, which fix planetary nodes within the solar system in accordance with spectral lines (Note 37), should also be manifested between stellar systems. Such a manifestation is found in the proportion

$$1^2 : (316.53+1)^2 :: 460 : 46,379,800 \ldots\ldots\ldots\ldots (1)$$

In this proportion, the mass at the centre of condensation (Earth) is represented by 1; the mass at the centre of nebulosity (Jupiter) by 316.53; the distance of *a Centauri* by 46379800; the distance of Earth's nascent locus (Note 45) by 460, or the cosmical equivalent of the corona line. The square of the mass varies as reciprocal *vis viva*, or orbital *vis viva* at uniform distance, and also as distance of orbital projection at uniform distance against uniform resistance.

The abscissas of the stellar-solar paraboloid may therefore be computed by means of Sun's semi-diameter, Laplace's limit, and the corona line, as follows:

$$v_\lambda = 214.45\, r_0 \div 497,827 = .4307721\, r_0 \ldots\ldots\ldots\ldots (2)$$

$$v_0 = 2\pi\, r_0 \times 214.45^{\frac{3}{2}} \div 31558149 = .0006252556\, r_0 \ldots\ldots (3)$$

$$g_0 = v_0^2 \div r_0 = .0000003909446\, r_0 \ldots\ldots\ldots\ldots (4)$$

$$t_r = 2v_\lambda \div g_0 = 2203750 \text{ sec} \ldots\ldots\ldots\ldots (5)$$

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*Seen near Brady's Mills, 15 miles east from Keyser.

$$\mathrm{L} = 214.45 r_0 \left(\frac{t_\mathrm{r}}{1 \text{ year}}\right)^{\frac{2}{3}} = 36.3658 r_0 \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\mathrm{M} = \left(\frac{v_\lambda}{v_0}\right)^2 r_0 = 474657.14 r_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$\log.\ \xi = \log.\ \frac{1}{6} = -.7781513$

$\log.\ \xi + \log.\ 19\eta + \log.\ 361\zeta = \log.\ (\frac{4}{9}\mathrm{L}) = 1.2085109 \ldots\ldots\ldots$

$\log.\ \xi + \log.\ 39\eta + \log. 1521\zeta = \log.\ [(317.53)^2 \times 460.002] = 7.6663302.$

$\therefore \qquad \log. \zeta = .0055982.$

$\log. \eta = -.0018046316.$

The following table gives the logarithms, both of the centripetal and the centrifugal abscissas, as calculated by the formula:

$$\log.\ \mathrm{A}_n = \log.\ \xi \pm n \log.\ \eta + n^2 \log.\ \zeta.$$

n	Centripetal, ($\log.\ \xi + n \log.\ \eta + n^2 \log.\ \zeta$).	Centrifugal, ($\log.\ \xi - n \log.\ \eta + n^2 \log.\ \zeta$)
o	—.7781513	—.7781513
1	—.7743577	—.7707485
2	—.7593678	—.7521492
3	—.7331814	—.7223536
4	—.6957986	—.6813616
5	—.6472195	—.6291731
6	—.5874439	—.5657883
7	—.5164719	—.4912071
8	—.4343036	—.4054294
9	—.3409388	—.3084554
10	—.2363776	—.2002850
11	—.1206200	—.0809182
12	.0063339	.0496451
13	.1444843	.1914047
14	.2938311	.3443607
15	.4543742	.5085132
16	.6261138	.6838620
17	.8090498	.8704072
18	1.0031821	1.0681489
19	1.2085109	1.2770869
20	1.4250361	1.4972213
21	1.6527576	1.7285522
22	1.8916756	1.9710794
23	2.1417900	2.2248030
24	2.4031007	2.4897231
25	2.6756079	2.7658395
26	2.9593115	3.0531523
27	3.2542114	3.3516616
28	3.5603078	3.6613672
29	3.8776006	3.9822692

	Centripetal.	Centrifugal.
30	4.2060898	4.3143676
31	4.5457753	4.6576625
32	4.8966573	5.0121537
33	5.2587357	5.3778413
34	5.6320104	5.7547254
35	6.0164816	6.1428058
36	6.4121492	6.5420826
37	6.8190131	6.9525559
38	7.2370735	7.3742255
39	7.6663302	7.8070915

In each set, the nine abscissas from 12 to 20 inclusive, and the nine from 30 to 38 inclusive, have no obvious* significance, except as indicating a symmetrical division of the abscissas, into three groups of 3^2 each, between the star and the Sun's surface. The nine from 21 to 29 inclusive indicate different planetary influences in the two sets, which seem to be worthy of further study. If we take the anti-logarithms and divide by 214.54, in order to get the abscissas in terms of Earth's semi-axis major, we find the following accordances :

n	Centripetal.	Planetary loci.		Centrifugal.	Planetary loci.	
21	.210	$\frac{1}{2}$ Mercury,	.199	.250	Mercury s. p.,	.297
22	.363	$\frac{1}{2}$ Venus,	.362	.436	‘‘ m. a.,	.456
23	.646	$\frac{2}{3}$ Earth m. p.,	.644	.782	Venus s. a.,	.774
24	1.180	$\frac{3}{4}$ Mars,	1.142	1.440	Mars m. p.,	1.403
25	2.209	$\frac{4}{5}$ Asteroid 55,	2.208	2.720	Asteroid 45,	2.720
26	4.246	$\frac{5}{6}$ Jupiter,	4.336	5.270	Jupiter,	5.203
27	8.374	$\frac{6}{7}$ Saturn,	8.176	10.479	Saturn s. a.,	10.343
28	16.943	$\frac{7}{8}$ Uranus,	16.786	21.382	Uranus s. a.,	20.679
29	35.178	$\frac{7}{6}$ Neptune,	35.040	44.765	$\frac{3}{2}$ Neptune m. p.,	44.598

The foregoing table furnishes the following indications :

1. That there are harmonic influences upon relative mass and position extending from the stars to the Sun, of a similar character to those which are manifested within the solar system.

2. That these are the only known and measurable evidences, which have yet been discovered, of inter-stellar action.

3. That the influences are of such a character as are required by the resistance of nodes of great comparative inertia in a highly elastic vibrating medium.

4. That there is, therefore, some inter-stellar medium, to which the laws of elastic action and reaction are applicable.

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* We find, however, upon examination, that log. A_{38}, 7.2370735)) is equal to log. L + log M, or log. LM, (1.5606934 + 5.6763800). This gives four elements for determining the paraboloid, where only three are required. Any three of the elements furnish data for determining the masses of Sun, Jupiter and Earth; Sun's volume and density; Laplace's limit; solar modulus of light; corona line; stellar distance and the rupturing planetary loci.

5. That the inter-stellar radial influences are so modified, by mutual action and re-action, as to become paraboloidal.

6. That the centripetal influences tend to produce and maintain nebular condensation, planetary segregation and orbital revolution.[*]

7. That the centrifugal influences tend to produce and maintain mean or apsidal positions for the condensed planets.

8. That the harmonic waves are propagated with the velocity of light.

9. That the controlling mass of the whole solar system (Sun), as well as the subordinately controlling masses of the extra-asteroidal and intra-asteroidal belts (Jupiter, Earth), are harmonically dependent upon the oscillations of the inter-stellar medium.

10. That the corona-, hydrogen-, and basic-lines serve to link cosmical with chemical, molecular, and other physical phenomena.

11. That the outermost known planet (Neptune) gives evidences of nebular influence, in its centrifugal as well as in its centripetal abscissa, the centripetal coefficient being the reciprocal of Saturn's, and the centrifugal coefficient being the reciprocal of Earth's centripetal. The centrifugal abscissa represents the locus of a nebular surface which would yield planetary aggregation in Neptune's orbit, through subsidence-collision.

12. That the centre of condensation (Earth) has no special centrifugal abscissa.

13. That the centrifugal abscissas for Venus and Mars point to early apsidal stages of central condensation.

14. That Earth's centripetal abscissa represents the locus of belt-formation in a nebula extending to Earth's mean rupturing locus (mean perihelion).

15. That the elements of the controlling paraboloid are all maxima, r_0, (Sun's semi-diameter), being the locus of greatest gravitating energy; L, (Laplace's limit), the limiting locus of greatest possible atmosphere; Jupiter and Earth, the greatest and controlling masses of their respective belts.

16. That the velocity of light and Sun's mass, upon which r_0 and L depend, are also maxima, and all physical phenomena within the system should be in harmonic dependence upon their mutual actions and reactions.

47. *Physical Units of Nature.*

In the Philosophical Magazine for May, 1881, Dr. Stoney republishes a paper on physical units, which he read before the British Association in 1874, giving approximate values for L_1, T_1, M_1. More than five years before his original paper was read, I had communicated to the American Philosophical Society[†] the equation, $u = g_1 t_1$, in which $u =$ velocity of light, which Stoney represents by $V_1 = 3$ VIII metres per second; $g_1 =$ acceleration of gravity at Sun's equatorial surface; $t_1 = \frac{1}{2}$ time of solar rotation.

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[*] See ζ, *antè*, pp. 205-6.

[†] Proc. A. P. S., xi, 106–7.

This equation furnished the data for computing a less arbitrary series of natural units, L_0, T_0, M_0. The British Nautical Almanac gives an estimate of Sun's apparent semi-diameter, (r_0), which makes Earth's semi-axis major equivalent to 214.45 r_0. Hence we find (see foregoing note):

$V_0 = V_1 = v_\lambda = .4307721\, r_0 = 3$ VIII metres.
$v_0 = .0006252556\, r_0 = 435443$ metres.
$G_0 = .0000003909446\, r_0 = 272.26$ metres.
$L_0 = \left(\frac{V_0}{v_0}\right)^2 r_0 = 474657.14\, r_0 = 33$ XIII metres = solar modulus of light.
$T_0 = \frac{V_0}{G_0} = 1101875$ sec.
$M_0 \propto gr^2 = 331700 \oplus = 2$ XXXIII grammes.

Among the apparent advantages of these units over those of Dr. Stoney, are the following:

1. L_0 is the height of a homogeneous æthereal solar envelope, which would propagate waves with the velocity of light.

2. T_0 is the time in which the sum of the solar centripetal accelerations of maximum gravitation is equivalent to the sum of the solar reactions against the centre of gravity of all other cosmical bodies, in the alternate projection from and return towards the disturbing centre of gravity.

3. M_0 is the controlling mass of the system, and, consequently, of all the physical phenomena which occur in any part of it.

4. L_0, T_0, M_0 can be estimated more easily and more accurately than L_1, T_1, M_1.

5. G_0 is the greatest possible gravitating acceleration within our system.

6. V_0 is the greatest known velocity of wave-propagation. It is true that Wheatstone's unconfirmed experiments, as well as my own ratio of *vis viva* of excursion to *vis viva* of propagation (9 : 5), indicate greater possible velocities. But Wheatstone did not eliminate the influence of induction, and Maxwell's application of my ratio was confined to the excursion of particles through very narrow limits.

7. All of those maxima are eminently *natural* maxima, and, therefore, suitable for natural standards.

The following equations show the mutual relations of the two sets of units:

$$L_0 \div L_1 = 33 \text{ L}.$$
$$T_0 \div T_1 = 33 \text{ L}.$$
$$M_0 \div M_1 = 2 \text{ XL}.$$

48. *Rotating Vis Viva.*

The photodynamic ratio of the velocity of light to the equatorial velocity of stellar rotation is expressed, as I have shown (Note 34), by the proportion

Stellar modulus of light : stellar semi-circumference : : v_λ : v_r.

The centre of gravity of the universe must be stable, so far as any modifying action of its constituent bodies is concerned. The particles of a star or nebula, exposed to æthereal oscillations, are solicited simultaneously towards the local and towards the universal centre of gravity. The accelerations, relatively to the stellar or nebular centre, on the side towards the universal centre, and the retardations on the opposite side, necessarily produce rotation. The duration of the rotation must be such as to maintain cyclical equilibrium among the three activities of general luminous undulation, general gravitation and local gravitation.

Equality of action and reaction should lead to harmonies of *vis viva* and of mass, which may become especially prominent when there are two largely preponderating bodies in a system, as in the case of Sun and Jupiter. Their mutual actions and reactions, being exerted through a common radius, are proportioned to their masses. The mean *vis viva* of rotation being represented by a virtual projection of mass through .4 r, if we consider the modulus of light as the virtual projection due to Sun's mass during the cyclical disturbances of equilibrium, we may suppose

Sun's mass : Jupiter's mass : : Modulus : .4 Jupiter's semi-axis major.
1047.879 : 1 : : 474657.14 : 453

Jupiter's semi-axis major is $5.2028 \times 214.45 = 1115.74$; .4 of $1115.74 = 446.3$.

The theoretical projection is, therefore, 1.0149525 times the mean-projection, and there is an exact accordance, twice during each revolution of Jupiter around the Sun. For, according to Stockwell,* the secular minimum value of Jupiter's eccentricity is equal to .0254928. Neptune's maximum eccentricity, according to the same authority, is .0145066, and $\frac{1}{3}$ of the mean eccentricity of Uranus is .0149538. These accordances seem significant, in view of Jupiter's central nebular position between Uranus and Neptune, at their opposition.

49. *Further Stellar Relations of Mass.*

The paraboloidal formula, $x_n = \xi \eta^n \zeta^{n^2}$, may be expressed under the form $x_n = \xi\ (\eta\ \zeta)^n\ \zeta^n$, the successive terms being found by the product of corresponding terms of two geometrical series. They may, therefore, be taken to represent the mutual actions and reactions of two co-ordinate masses, like Sun and Jupiter. The geometric variations of density, which accord with arithmetical variations of distance, suggest the proportion, (Note 46):

Sun's mass : Jupiter's mass : : x_{39} : 39 × Jupiter's semi-axis major.
1047.879 : 1 : : 46379946† : 44260.8

This gives, for Jupiter's theoretical semi-axis major, $44260.8 \div 39 = 1134.9$, which is 1.017165 times the estimate of the British Nautical Almanac (Note 48). This is less than the secular minimum aphelion of Jupiter, so that the locus is traversed twice during each revolution of Jupiter about the Sun.

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* Smithsonian Contributions, 232, p. 38.

† The *logarithm* of x_{39} being 7.6663302, the abscissa itself is 46379946 r_0.

If we make the greatest possible allowance for uncertainties of data, the relations of inter-planetary and inter-stellar distances to the three important masses, which are shown by the present note and the three foregoing notes, are close enough to be very curious, even if we could think them merely accidental. Inasmuch as they have been discovered by a simple consideration of the consequences which ought to follow, through the known laws of elasticity, from the interruption of luminous vibrations and the formation of nodes by cosmical inertia, they are more than curious, for they are indicative of the unity of energy which has long been thought to control all physical manifestations.

50. *Harmonic Masses.*

Twice modulus is the radius of rectilinear oscillation, which would be synchronous with orbital oscillation about a major axis equivalent to that of the homogeneous æthereal atmosphere. The dependence of modulus upon solar mass and radius makes it probable that there may be a harmonic dependence, of a like character, among the principal planetary masses. We find, indeed, that the four extra-asteroidal planets have harmonic masses, inasmuch as they may all closely be represented by the formula $\frac{\text{Sun}}{\text{Planet}} = \frac{2 \times \text{Modulus}}{n\, r_0}$, in which n is integral, so that the second member of the equation gives four terms of a harmonic progression.

$n\, r_0$	$2\text{M} \div n\, r_0$	Computed Values.	Authority.
$906\, r_0$	1047.81	1047.88 = Sun ÷ Jupiter.	Bessel.
$271\, r_0$	3503.	3501.6 = Sun ÷ Saturn.	"
$42\, r_0$	22603.	22600. = Sun ÷ Uranus.	Newcomb.
$49\, r_0$	19374.	19380. = Sun ÷ Neptune.	"

That these approximations are not accidental, may be shown by comparing the probable with the actual deviations from exact accordance.

Probable.	Actual.	P ÷ A.
$\frac{1}{4}$ of $\frac{1}{906} = .00027$	$7 \div 104781 = .000067$	4
$\frac{1}{4}$ of $\frac{1}{271} = .00092$	$14 \div 35030 = .000399$	2.3
$\frac{1}{4}$ of $\frac{1}{42} = .00595$	$3 \div 22603 = .000133$	46
$\frac{1}{4}$ of $\frac{1}{49} = .00510$	$6 \div 19380 = .000309$	17

The deviations are so far within the limits of probable errors of mass that the accordance may, perhaps, be exact. But absolute exactness, in the satisfaction of any mechanical tendencies is not to be expected. There are so many and so various modifying influences, in all cases, that any result which is in complete accordance with theory should be regarded as more suspicious than one which suggests slight perturbations from some unknown cause.

51. *Subordinate Masses.*

If "the exception proves the rule," we may find that the perturbations, which were spoken of in the foregoing note, may introduce apparent exceptions, in subordinate masses, which are really confirmatory of the harmonic tendencies in larger masses. Taking the nearest integral values of n for each of the intra-asteroidal planets, we may construct the following comparative table :

$n\, r_0.$	$2\,M \div n\, r_0.$	Computed Values.	Authority.
1	959314	3093500 = Sun ÷ Mars.	Hall.
3	319771	331700 = Sun ÷ Earth.	Chase.
2	479657	427240 = Sun ÷ Venus.	Hill.
1	959314	4865751 = Sun ÷ Mercury.	Encke.

The actual values of nr_0, which would give the computed values of mass, are .307, 2.862, 2.222, .204. There is an approximation to the integral values of n, in the cases of Earth and Venus, that of Earth having the same probability as that of Saturn, 2.3. The probability for Venus is only ½ as great, or 1.15. The masses of Mercury and Mars are evidently controlled by some other law, which overrides that of the repetition of solar radial nodes.

52. *Reciprocal Coefficients.*

I have pointed out* the reciprocity between the coefficients of the stellar-solar abscissas which indicate rupturing loci for Saturn and Neptune. This reciprocity is interesting, not only for indicating the inverse rotation of two portions of a primitive nebula, but also for introducing the harmonic ratio of mass, into the retrograde portion, which is shown in Note 50. We accordingly have the two following proportions :

$$x_7 : \rho_7 :: m_8 : m_9 :: 6 : 7.$$
$$x_9 : \rho_9 :: m_9 : m_8 :: 7 : 6.$$

In these proportions, x represents the stellar-solar abscissa, or rupturing locus ; ρ, semi-axis major ; m, mass ; subscript $_7$, $_8$, $_9$, Saturn, Uranus, Neptune.

53. *Total Planetary Mass.*

Notes 46, 48, 49 and 52 indicate the probability of some photodynamic influence in determining the primitive ratio of solar (m_0) to planetary mass (m_n). On examination we find

$$\tfrac{7}{6}\text{ modulus} : \text{Sun's semi-diameter} :: m_0^2 : m_n^2.$$
$$553767 : 1 :: 744.155^2 : 1^2.$$

The following table gives the computed masses of the several planets, Sun's mass being represented by 10,000,000,000.

Mercury,	2,055	Encke.
Venus,	23,406	Hill.
Earth,	30,148	Chase.
Mars,	3,233	Hall.
Jupiter,	9,543,087	Bessel.
Saturn,	2,855,837	Bessel.
Uranus,	442,478	Newcomb.
Neptune,	515,996	Newcomb.
Sum,	13,416,240 = $m_0 \div 745.365.$	

**Ante*, p. 209.

This value differs by less than $\frac{1}{6}$ of one per cent. from the above theoretical value. If the sum of the asteroidal and cometary masses is about $\frac{2}{3}$ as great as Earth's mass, the accordance is exact. The additional evidence which it furnishes, both of early photodynamic rupturing and reciprocally rotating influences, ($\frac{7}{6}$, $\frac{6}{7}$), may prove to be as important as it is interesting. The total planetary mass is very nearly 2 × 7 times the retrogradely rotating mass of Uranus and Neptune.

54. *Atomic Weights.*

The close approximation to integral mass-divisors (Notes 50, 51), in all the known primary planets except Mercury and Mars, is like the atomic approximations to exact multiples of the hydrogen unit. This fact, together with the many evidences of tendencies to harmonic wave-lengths among spectral lines, suggests the probability of harmonic influences upon atomic weight, as well as upon atomic velocity.

There is still so much uncertainty in regard to many of the atomic weights, that it is impossible to determine, with exactness, the various subdivisions of which any given element may be susceptible. It is evident, however, that certain factors enter more frequently into the atomic ratios than could be looked for from the mere laws of probability, and it does not seem unreasonable to hope for future light upon the nature of chemical combination, from the study of mathematical combinations.

The factors which seem worthy of special attention are, 2, 2^2, 2^3, 2^4, 3^2, 5^2, 7, 13. The combination of the powers of 2 with prime numbers is shown in the following table :

	1	3	5	7	11	13
2	—	—	—	N	—	—
4	—	C	—	Si	Sc	Cr
8	—	Mg	Ca	Fe	Y	Ru, Rh
16	O	Ti	Br	Cd	—	—
32	S	Mo	—	—	—	—

The following groups are also suggestive :

4 × 3	C	4 × 22	Y	7 × 1	Li	7 × 26	Ta
4 × 4	O	4 × 24	Mo	7 × 2	N	7 × 30	Bi
4 × 6	Mg	4 × 26	Ru, Rh	7 × 4	Si		
4 × 7	Si	4 × 27	Ag	7 × 5	Cl	13 × 3	K
4 × 8	S	4 × 28	Cd	7 × 8	Fe	13 × 4	Cr
4 × 10	Ca	4 × 35	Ce	7 × 9	Cu	13 × 5	Zn
4 × 11	Sc	4 × 45	La	7 × 10	Ga	13 × 6	Se
4 × 12	Ti	4 × 46	W	7 × 16	Cd	13 × 8	Ru, Rh
4 × 13	Cr	4 × 50	Hg	7 × 18	Te	13 × 14	Ta
4 × 14	Fe	4 × 51	Tl	7 × 19	Cs	13 × 15	Os
4 × 17	Ga	4 × 52	Bi	7 × 20	Ce	13 × 16	Bi
4 × 20	Br	4 × 60	U				

Bismuth, on account of the uncertainty of its equivalent, appears in each group. If 208 is its true value, it should be excluded from the first two groups. All these arrangements are, for the present, only tentative, but I think that an increase in accuracy of determination will be more likely to increase than to diminish the number of indications of this kind.

My anticipation of a "ponderable disturbance of equilibrium, which must give rise to æthereal oscillations in every chemical action," was published in 1864*. The confirmations of that anticipation, by Cooke, Mendelejeff, Meyer and others, lead me to hope that a careful consideration of the common factors in different supposed elements will help towards further analysis and, perhaps, towards the final discovery of the laws by which all chemical substances are developed from a single primitive form of matter.

55. *A Natural Thermal Unit.*

Any expression for the mechanical equivalent of heat which introduces the degrees of a thermometer-scale, must have an arbitrary character which may interfere with the ready conversion of thermal into other dynamic measurements. Moreover, it may well be questioned whether the mass-factor should be chosen, rather than the velocity-factor of *vis viva,* as a unit of comparison. If we consider force as a representative of matter in motion, different kinds of force would seem to be fitly characterized by differences of velocity rather than by differences of mass.

There is an obvious propriety in taking water as a representative medium, but there is no more need of considering a pound, or a kilogram, or any other definite quantity, in ordinary thermodynamic investigations, than there is in chemical researches. The three changes of state through which water passes, represent different well-defined amounts of work. The changes from gas to vapor and from liquid to solid do not involve any essential change of form, but the change from vapor to liquid converts an indefinitely expansible and compressible substance into one which is but slightly expansible and almost incompressible. It thus furnishes a natural thermal unit (v_k), which is a bond between tendencies to aggregation and to dissociation and a convenient standard for comparison between the equal actions and reactions of various forms of energy.

An atom or quantity x of water, in passing from the fusing to the boiling temperature, absorbs an energy sufficient to lift itself 772×180 feet, or 423.54×100 metres. This represents a wave- or projectile-velocity $v_k = \sqrt{2\,gh} = 2986.3$ feet $= .56558$ miles $= .9102$ kilometres per second.

56. *Weighing the Sun by the Thermometer.*

In two wave-systems, which are due, either directly or indirectly, to synchronous actions and reactions between two different masses, the wave-velocities are proportioned to the producing masses; for $v_k \propto gt \propto \frac{\mu\, t}{r^2} \propto \mu$. We have already found (Note 47) that the wave-velocity of solar reaction against æthereal action ($G_0 T_0$) is the velocity of light (v_λ). Therefore, if v_λ and v_k are primitively synchronous, designating Earth's semi-axis major by ρ_3,

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*Proc. Amer. Phil. Soc., ix, 439.

$$\text{Sun's mass } (\mu_0) : \text{Earth's mass } (\mu_3) : : v_\lambda : v_k : : \frac{\rho_3}{497.827} : .56558 \text{ miles} \,.\,. (1)$$

If we designate Sun's and Earth's densities, respectively, by δ_0 and δ_3.

$$\mu_0 : \mu_3 : : \left(\frac{\rho_3}{214.45}\right)^3 \delta_0 : r_3^3 \, \delta_3 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\delta_3 : \delta_0 : : \left(2\pi\sqrt{\frac{r_0}{g_0}}\right)^2 : \left(2\pi\sqrt{\frac{r_3}{g_3}}\right)^2 : : (10049)^2 : (5073.6)^2 : : 1 : .2549 \,.\,. (3)$$

Substituting (3) in (2) we find, by elimination between (1) and (2),

$$\mu_0 = 328{,}424 \; \mu_3.$$
$$\rho_3 = 92{,}472{,}100 \text{ miles.}$$
$$r_0 = \rho_3 \div 214.45 = 431{,}206 \text{ miles.}$$

57. *Convertibility of Natural Units.*

Stoney's notation, with the modifications proposed in Note 47, connects the electric and photic units through the equation :

$$v_e = L_0 \div T_0 = I \div i = e \div E = (C \div c)^{\frac{1}{2}} = (r \div R)^{\frac{1}{2}}.$$

The gravitating, photic, thermal and electric units are, therefore, interconvertible by the equation

$$G_0 \, T_0 = v_\lambda = \frac{\mu_0}{\mu_3} v_k = v_e = 185{,}751.6 \text{ miles} = 298{,}930 \text{ kilometres.}$$

It seems probable, in view of the mean result of my other physical investigations, that the above values of μ_0 and ρ_3 may be, respectively, about one per cent., and $\frac{1}{3}$ of one per cent., too small. If the calorie and J were estimated at the equator, the approximation would probably be still closer and, perhaps, exact.

58. *Solar Temperature.*

The infinitely projectile or radiating power of a central mass varies as $\sqrt{2\,gr} \propto \sqrt{\frac{\mu}{r}}$. When the comparison of mutual solar and terrestrial actions and reactions is made through their common radius or semi-axis major, $\sqrt{\frac{\mu}{r}} \propto \sqrt{\mu} \propto \sqrt{\frac{v_\lambda}{v_k}}$. As v_k is the thermal equivalent-velocity at Earth's surface for 100° C., we may infer that the mean radiating temperature, at Sun's surface, is $g_3 \times 100^\circ \times \sqrt{328{,}424} = 27.739 \times 57308^\circ = 1589590^\circ$ C. The temperature deduced from Sun's semi-diameter (r_0) is $\frac{r_0}{1389.6 \text{ ft.}} = 1638430^\circ$ C., being 1.0307 times the value which is indicated by the mutual actions and reactions of solar and terrestrial mass. This approximation would be very satisfactory, even without attempting to account for the discrepancy. It is of the same order of magnitude as the orbital projections of the planets, and it represents, very closely, the *vis viva* of Jupiter's secular apsidal oscillation. For the ordinary thermal equivalent varies as the square of the corresponding velocity, or as the fourth power of orbital velocity. The fourth power of 1.0307 is 1.1287 ; Jupiter's secular *vis viva* of projection, according to Stockwell,* is $\frac{1.06083}{.93917} = 1.1296$.

59. *Permanency of Standards.*

The three controlling masses, at the nebulous centre, the nucleal centre, and the centre of condensation, exert a combined thermodynamic influence which may lend interest to a closer examination of the several interconvertible standards (Note 57). It is noteworthy, at the outset, that they are all maxima ; G_0 representing the greatest gravitating energy in the solar system ; T_0, the cyclical time of maximum disturbance of the centripetal tendencies of G_0 ; $v_\lambda = v_e$, the greatest known velocity of wave-propagation ; μ_0, the greatest mass in the solar system ; μ_3, the mass which exercises the greatest local influence upon the physical phenomena which are susceptible of the most minute observation ; v_k, the greatest thermal range between the centripetal and centrifugal forms of the most widely diffused and most important body which is well known in the three states of solid, liquid and gas ; $L_0 = G_0 T_0^2$, the sum of the accelerations of G_0 during T_0. The law of conservation of areas, as well as the law of constancy, requires that G_0 should vary inversely as T_0, in all stages of solar nebular condensation. The photic unit, v_λ, on account of the abundant evidence of its universal activity, as well as on account of the laws of equal action and reaction, and of the inverse variability of elasticity and density in homogeneous media, is presumably invariable. The thermal unit, v_k, is practically constant, but its dependence upon relations of mass which may be subject to slight, though inappreciable, variations, subordinates it to the photic unit. The electric unit, v_e, being identical with the photic unit, the question arises, which of the two should be regarded as primary, and which as secondary. Do not the facts that light is universal, while the various electrical units represent local relations, indicate the proper answer?

60. *Heat and Electricity.*

The probability of arriving at the unity of force through the study of æthereal oscillations, was indicated by me in 1864.† The special relations of electricity to heat, which Edlund subsequently discussed with great ability and fullness, are shown by the proportions,

$$\left(\frac{I}{i} = \frac{e}{E} = \sqrt{\frac{C}{c}} = \sqrt{\frac{r}{R}} = \frac{L_0}{T_0}\right) : v_k : : \mu_0 : \mu_3.$$

$$C : c\, v_k^2 : : r : R\, v_k^2 : : \mu_0^2 : \mu_3^2.$$

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* Smithsonian Contributions, 232, p. 38.

† Proc. Am. Phil. Soc., ix, 356-60.

61. *Photo-thermal Unit.*

Since temperature varies as the square of the velocity, we have,

$$v_k^2 \ : \ v_\lambda^2 \ :: \ 100^\circ \ : \ x.$$

$$.56558^2 : 185{,}751.6^2 :: 100^\circ : 10786240000000^\circ.$$

The corresponding projectile abscissa may be found, in terms of Sun's semi-diameter, by multiplying by 1389.6 and dividing by r_0, (Note 56); $1389.6 \times 10{,}786{,}240{,}000{,}000 \div (431{,}206 \times 5280) = 6583265$; log. $6583265 = 6.8184413$. This agrees, within less than $\frac{1}{7}$ of one per cent., with the photodynamic determination of the third centripetal abscissa in the stellar-solar paraboloid, (log. $A_{37} = 6.8190131$, Note 46).

62. *Thermo-gravic Paraboloid.*

By taking $\frac{1}{6} r_0$, $\frac{4}{9}$ L, and A_{37} as the paraboloidal elements, we substitute the thermal for the photic aspect of the æthereal waves and are enabled to compare gravitating actions with thermal reactions. We thus find,

$$\text{Log. } \eta = -.001788316$$

$$\text{Log. } \zeta = .0055973414$$

The closeness of agreement between the photic and thermal abscissas is shown by the following comparisons:

		Locus.	Photic.	Thermal.	Difference.
A_{21}	$\frac{1}{2}$	Mercury,	1.6527576	1.6527216	.0084 of 1 per cent.
A_{29}	$\frac{7}{6}$	Neptune,	3.8776006	3.8773517	.0578 " "
A_{39}	α	*Centauri,*	7.6663303	7.6656606	.156 " "

The greatest difference is, therefore, less than $\frac{1}{6}$ of one per cent. The photo-thermal unit (A_{38} = L M, see Note 46), is intermediate between the photo-gravic unit (A_{37}), and the stellar locus (A_{39}). We have thus five elementary loci where only three are required. By equations of condition we ought to be able to determine all the values which are involved, with much greater accuracy than will ever be possible by astronomical observations.

63. *Unity of Force.*

The belief that all forms of force are modifications of one fundamental energy, is as old as the oldest Greek philosophy, but no approximation towards a demonstration of the belief seems to have been made until Rumford performed his experiments at Munich, in 1798. During the present century, the undulatory theory of light has commanded a general acceptance, the science of thermodynamics has been largely developed, and Tyndall's works have made the reading world familiar with the proofs that heat is a mode of motion. Stephenson taught that the energies of sunlight were stored in vegetable growths and in coal-beds; Herschel called attention to the enormous elasticity of luminiferous æther and to the Sun's rays as "the ultimate source of almost every motion which takes place on the surface of the earth;" Titius, Bode, Alexander and Kirkwood, presented evidences of law in planetary arrangement; Peirce and Hill found similar evidences in times of planetary revolution; Faraday strove in vain to reconcile the law of gravitation with the theory of conservation of energy; Weber and Kohlrausch introduced a general kinetic notation and found the importance of the velocity of light in electrical phenomena; Faraday discovered the magnetic influence upon light and Sir William Thomson stated that "the explanation of all phenomena of electromagnetic attraction or repulsion, and of electromagnetic induction is to be looked for simply in the inertia and pressure of the matter of which the motions constitute heat;" Maxwell* considered magnetic force as the effect of the centrifugal force of vortices, electromagnetic induction as the effect of the forces called into play when the velocity of the vortices is changing, electromotive force as arising from the stress on the connecting mechanism, and electric displacement as arising from the elastic yielding of the connecting mechanism.

64. *Reclamation.*

In 1863, I began a systematic series of investigations, in the hope of *verifying, by numerical measurements,* some of the supposed consequences which seemed to flow from the foregoing researches and hypotheses, and of *finding new clues* to the values and relations of the fundamental elements of physical energy. The division of forces into attractive and repulsive, as well as the equality of action and reaction led me to the study of the laws of elasticity and their influence on undulatory and cyclical motions. In my first paper† I called attention to the effects of cyclically alternating acceleration and retardation during rotation or revolution. In the course of the seven following months, I pointed out various relations which seemed to have been previously unnoticed, between heat, gravitation, electricity and magnetism, showing that many of the phenomena could be imitated and explained by the mechanical propagation of vibrations. The tendencies to equilibrium between gravitating pressure and æthereal elasticity, the necessary production of continual oscillations by opposing forces, the possibility of accounting for gravitation, electricity and magnetism by such oscillations moving with the rapidity of light, were all discussed during this period.

65. *Musical Rhythm.*

Fourier (Note 33) seems to have first given a mathematical demonstration of the *necessity* that cyclical motions should be also rhythmical. If the luminiferous æther is anything more than a hypothesis, which helps to explain an indefinite variety of phenomena, the tendency to the production of harmonic nodes in elastic media must greatly multiply the rhythmical manifestations, both cosmical and molecular. Some of those harmonies have long been known, their discovery having been apparently the result of accident, or of a happy guess. The number which will hereafter be made known will depend, in large measure, on the skill with which lines of investigation are guided by an understanding of the influences and consequences of elastic action and reaction.

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* Electricity and Magnetism, ii. 417.

† Proc. Am. Phil. Soc., ix, 285, Dec. 1863.

66. *Mass, Distance and Time.*

Laplace regarded force as proportional to velocity, treating momentum, or the product of mass by velocity, as the result of force acting on mass. He defined velocity as "the ratio of the space to the time employed in describing it." More recent writers consider force as equivalent to *vis viva*, involving both matter and motion, and represented by one-half the product of mass by the square of velocity, $\frac{mv^2}{2}$. Often, however, they follow Laplace in treating velocity as a ratio, designating distance or length by l, and time by t. The symbols m, l, t, therefore, are applicable in all discussions of force, and their use has paved the way for a ready acceptance of any evidence which may be adduced respecting the substantial unity of force. The fluctuating and arbitary values which have been assigned to the symbols, and the lack of any universally recognized standards of measurement, have obstructed the discovery of evidence without weakening the belief in kinetic unity. Even the recognition of a common velocity, in light and electricity, seems to have led to no systematic search for common standards of length and time.

67. *Maxima, Minima and Means.*

In all ordinary investigations, in any special department of dynamics, we deal with observed values which have a limited range, with a view of finding the most probable mean. In the study of forces which vary in accordance with known or supposed law, minimum or vanishing values claim special consideration. For a full understanding of any force we must also know the greatest energy of which it is capable, or of which we can obtain any practical experience. If we wish to compare different kinds of force, in order to ascertain the character of supreme and controlling energy, it seems especially important that we should turn our attention, at the outset, to the greatest manifestations of each of the elements of energy, m, l, t.

68. *Light and Electricity.*

For a long time, little was known of electricity except the phenomena of attraction, repulsion, and luminous manifestation. Faraday's want of success in searching for some bond of union between electricity and gravitation, deterred others from continuing the search. The undulatory theory of light enabled Weber and Kohlrausch to make their first determination of a velocity, which serves to connect many electrostatic and electromagnetic phenomena, and which is of the same order of magnitude as the velocity of light, v_λ. Their subsequent investigations, together with those of Thomson, Maxwell, Ayrton and Perry, showed that the two velocities are not only of the same order of magnitude, but that they are probably identical, so that light and electricity may be properly regarded as different modifications of a more fundamental energy. Their common velocity is the greatest known velocity of wave-propagation, and we are thus further encouraged to the study of maxima as indications of kinetic unity. The elements l, t, and m are still arbitrarily assumed in electrical treatises, notwithstanding the suggestiveness of the well-known equation of wave-velocity, $v = \sqrt{2\,gh}$.

69. *Electrostatic and Electromagnetic Dimensions.*

Although arbitrary values have been assigned to m, l and t, in electrical discussion, the same values are generally assumed in electrostatic and electromagnetic comparisons. It has thus been found that the ratio between the two systems is the velocity of light for quantity of electricity, electric current, magnetic potential, electric displacement, surface density, magnetic force, strength of current at a point, quantity of magnetism, electrokinetic momentum of a circuit, line-integral of electromotive force, magnetic induction, electromotive force at a point, and vector potential. The ratio is the square of the velocity for electrostatic or electromagnetic capacity, dielectric or magnetic inductive capacity, conductivity and resistance. The product of these two ratios by mass, furnishes indications of momentum and *vis viva* which will doubtless be useful in many general kinetic comparisons.

70. *Heat and Gravitation.*

The direct opposition of tendencies in heat and gravitation, the former being centrifugal and the latter centripetal, leads to an easier and more direct means of comparison between the two energies than has been found in the case of light and electricity. But the adoption of mh or $\frac{ml}{2}$, as the mechanical equivalent of heat, leaves the relation of gravitation out of view, and it is not readily seen until we find that $mh = \frac{2\,m}{g} \times \left(\frac{h}{t}\right)^2$. Even then, the foot-pound and second, or the kilogrammetre and second, are such arbitrary values that there is no indication of any uniform standard like the velocity of light. We see, on further examination, that our knowledge of heat is wholly derived from phenomena at the Earth's surface, where terrestrial gravitation is a maximum, and we are thus led to accept Earth's mass as a fitting natural unit of mass in thermodynamic questions, especially in all questions which involve the factor $\frac{2\,m}{g}$. In order to reach kinetic unity, some values should be found for l and t which can be presented under some simple relation to the velocity of light.

71. *Central Energy of the System.*

We are now somewhat prepared to appreciate the reasoning of Herschel and Stephenson, in referring all terrestrial energies to the Sun, and to accept M_0 and G_0, or Sun's mass and superficial gravitation, as standards of comparison between Sun's centripetal and centrifugal, or gravitating and radiating energies. The Sun itself is subject to the paramount control of universal mass and gravitation. Although we have no means of finding their absolute or relative value, we may symbolize them by M_u and G_u and thus seek for standard values of L_0 and T_0. Every particle of the Sun is continually solicited by G_0 and G_u, and their varying solicitations can be compensated only by cyclical alternations of rise and fall, from and towards the universal centre of gravitation, c_u. In order that Sun's mass may be stable, the sum of the tendencies of G_0 must be equal to the sum of the tendencies of G_u, during each half-rotation. A solar half-rotation, therefore, may be taken as a natural standard value for T_0; then the equation $M_0 H_0 = \frac{2 M_0}{G_0} \times \left(\frac{H_0}{T_0}\right)^2$ gives $H_0 = \frac{G_0 T_0^2}{2}$, and $L_0 = 2 H_0 = G_0 T_0^2$ as a corresponding natural standard of length and distance.

72. *Standard Kinetic Modulus.*

If we examine the equation which we have found in the foregoing note for the natural unit of length, we find that H_0 represents the height of virtual fall which would give a velocity equivalent to $G_0 T_0$, and L_0 is the height of a homogeneous elastic atmosphere which would have a velocity of wave-propagation equivalent to $\frac{L_0}{T_0} = G_0 T_0 = G_u T_0$, the actions and reactions of G_0 and G_u being equal. But the elastic atmosphere, in which these actions and reactions take place, is the universal æthereal medium, and its velocity of wave-propagation, $\frac{L_0}{T_0}$, is the velocity of light. The relations of heat, mass and gravitation, therefore, lead us to the same fundamental velocity as the relations of the electrostatic and electromagnetic systems, and the same velocity as the solar radiations which are variously modified so as to produce the phenomena of light, heat, electricity, magnetism and actinism. Our knowledge of the significance of the electrical unit is thus extended, so as not only to include the ratio $\frac{L_0}{T_0}$, but also to show that the natural standard units of mass, length, and time are, respectively, $M_0 =$ Sun's mass; $L_0 =$ standard kinetic modulus, or Solar modulus of light; $T_0 =$ half-time of solar rotation, or time of cyclical equality of action and reaction between solar inertia and æthereal undulation.

73. *Variable Units.*

In comparing arbitrary and variable units with the maximum standards, we have generally to deal with fractional values. In uniform velocities, like those of light, electricity, and primitive kinetic radiation, $l \propto t$; in variable velocities which are due to centripetal or centrifugal radiation, like those of gravitation and heat, $l \propto t^2$. Maxwell* gives electric dimensions and formulas which can be readily converted into standard kinetic units ; gravitating formulas can be similarly converted through the correlations, $g \propto \frac{m}{l^2} \propto \frac{l}{t}$, $\sqrt{g r} \propto \left(\frac{m}{l}\right)^{\frac{1}{2}}$; thermal formulas, through the correlations, $w \propto [m]\, h \propto [m]\, v^2 \propto [m]\, \frac{l^2}{t^2}$.

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*Op. cit., ii, § 620–9.

74. *Complex Variability.*

Any variable unit may be dependent upon some other variable unit, and different values of m, l and t may thus be involved in the same expression. It is necessary, therefore, that the significance of every variable symbol should be fully understood before undertaking to convert it into terms of the corresponding standard unit. For example, in the thermal and electric formulas, $[m]$ refers to the mass which is lifted against the constant pull of gravity, while in thermal formulas v, l and t also introduce considerations which are dependent upon the centripetal energy of Earth's mass, m_3, at a constant distance. The conversion of thermal into standard formulas, therefore, requires due attention to the ratios both of $[m]$ and m_3, to M_0. In such cases it may often be well to introduce a second natural unit, which can be simply derived from the standard unit, as in the deduction (Note 56) of the terrestrial thermal unit of velocity from the solar unit of luminous velocity, $v_k = \frac{m_3}{\mathrm{M}_0} \mathrm{V}_0 = m_3 \mathrm{M}_0^{-1} \mathrm{L}_0 \mathrm{T}_0^{-1}$.

75. *Notation and Approximate Values.*

Let $r_a =$ Sun's apparent semi-diameter $\doteq$* $961.''83$........2.9830990
$n = r \div r_a \doteq 214.45$..............................2.3313261
$r_0 =$ Sun's semi-diameter $\doteq$ 431,208 miles.............5.6346866
$\rho_3 = n r_0 =$ Earth's semi-axis major $\doteq$ 92,472,500 miles..7.9660127
$r_3 =$ Earth's semi-diameter $\doteq$ 3962.8 miles............3.5980022
$p_0 =$ Sun's parallax $= \frac{r_3\, r_a}{r_0} \doteq 8.''8392$................ .9464146
$t_0 = 2\pi\,(r_0\, \mathrm{G}_0^{-1})^{\frac{1}{2}} = 1$ year $\div\, n^{\frac{3}{2}} \doteq$ 10049 seconds...4.0021223
$t_3 = 2\pi\,(r_3\, g_3^{-1})^{\frac{1}{2}} \doteq 5073^{\mathrm{s}}.6$.........................3.7053158
d_0, $d_3 =$ density of Sun, Earth.......................
$\frac{d_0}{d_3} = \left(\frac{t_3}{t_0}\right)^2 \doteq .25491$..............................$\bar{1}.4063870$
$l_0 =$ Laplace's limit $= n\left(\frac{2\,\mathrm{T}_0}{1\ \mathrm{yr.}}\right)^{\frac{2}{3}} \doteq 36.3658\, r_0$........1.5606934
$\mathrm{V}_0 = \frac{\mathrm{L}_0}{\mathrm{T}_0} = \rho_3 \div 497.827 \doteq .4307721\, r_0 =$ velocity of light.$\bar{1}.6342476$
$v_0 = 2\pi\, r_0 \div t_0 \doteq .0006252556\, r_0$....................$\bar{4}.7960576$
$v_k = (2\, g_3 \times 180\ \mathrm{J} \div 1\ \mathrm{lb.})^{\frac{1}{2}} \doteq .56558$ miles..........$\bar{1}.7524940$

**Nearly* equal. The right hand column contains the approximate logarithms.

$g_3 = 32.088$ ft. $\doteq .0060773$ miles.......................$\bar{3}.7837087$
$G_0 = v_0^2 \div r_0 \doteq .0000003909446\, r_0$.....................$\bar{7}.5921152$
$T_0 = V_0 \div G_0 \doteq 1101875$ seconds.....................6.0421324
$L_0 = V_0\, T_0 = (V_0 \div v_0)^2\, r_0 \doteq 474657\, r_0$................5.6763800
$M_0 = V_0 \div v_k = r_0^3\, d_0 \div r_3^3\, d_3 \doteq 328438\, m_3$............5.5164402

Substituting the value of r_0 we find,

$l_0 \doteq$ 15,681,230 miles.............................. 7.1953800
$V_0 \doteq$ 185,752 miles................................. $\bar{5}.2689342$
$v_0 \doteq$ 269.615 miles 2.4307442
$G_0 \doteq$ 890.1 ft. $\doteq$ .16858 miles........................ $\bar{1}.2268018$
$L_0 \doteq 2213.37\rho_3 \doteq 73.67$ Neptune's semi-axis major $\doteq$
204,675,900,000 miles........................11.3110666

76. *Vis viva of Wave Propagation.*

In 1857, Professor Stephen Alexander announced to the American Association the approximate equality of md^2 in the two planets, Jupiter and Saturn, which make more than $\frac{12}{13}$ of the aggregate planetary mass in our system. He also showed that the mean distance of Saturn is nearly equivalent to $\frac{9}{5}$ of Jupiter's mean distance, and that the ratio $\frac{9}{5}$ has been largely influential in planetary distribution. In 1872, I showed* that the same ratio may be deduced from the thermal energy of chemical combination, representing the ratio of mean *vis viva*† of oscillating particles to *vis viva* of wave propagation. In 1877, Maxwell and Preston‡ published the same ratio, without seeming to have known that I had already deduced it from the laws of kinetic oscillation in elastic media. Its importance in discussions of kinetic unity, is evident from the fact that it is alike operative in the establishment of cosmical§ and of molecular harmonies.

77. *Conical Pendulums.*

The tendencies to concentration, through the mutual attractions of particles which are subjected to a simultaneous linear projection, and other combinations of tendencies to uniform and to variable velocities, lead to many oscillations which follow the laws of conical pendulums. It is therefore important, in investigations which involve cyclical elastic action, to inquire what forms of pendulum-vibration will best fulfil the requirements of Fourier's theorem. The general formula for the velocity of a conical pendulum is,

$$v = \sqrt{gr \sin\theta \tan\theta}.$$

Hence, since $t = 2\pi r \sin\theta \div v$, we find

$$t = 2\pi \sqrt{\frac{r}{g} \tan\theta}.$$

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* Proc. Am. Phil. Soc., xii, 392–4.
† Ib. foot note.
‡ P. Mag. [5], iii, 453; iv, 209.
§ Proc. A. P. S., xii, 403–5, *et seq.*

This is equivalent to twice the time of vibration in a simple pendulum of the same height, ($\cos\theta$), or to the time of vibration in a linear pendulum of four times the height. The requirements of *vis viva*, synchronism, and the conversion of wave-propagating velocity into velocity of oscillation under central action, are all satisfied, as I have shown in my discussions of explosive energy (Note 16, *et al.*), by combining the ratios of this and the foregoing note, $\frac{1}{4} \times \frac{5}{9}$.

78. *Standard Temperatures.*

In thermodynamic deductions and comparisons, it is often important to decide what temperature should be taken as the standard. Whenever a sufficient number of kinetic values have been deduced for the natural units, M_0, L_0, T_0, the introduction of conditioning equations may help us to much accurate and useful knowledge which will lead to greater precision in many details of molecular physics. The present range of uncertainty may be estimated by examining the influence of two different possible standards upon the results of Note 16. The combining calories appear to have been observed, in each instance, at the temperature of 15° C. To reduce the mean to 0°, we have

$$\begin{array}{ll} H_2 = 2 \times 15 \times 3.409 = & 102.27 \\ O = 16 \times 15 \times .2175 = & 52.20 \\ \hline \text{Total deduction} = & 154.47 \end{array}$$

Leaving 68886 — 154.47 = 68731.53 calories

$$\tfrac{5}{9} \times \tfrac{1}{4} \times \tfrac{1}{9} \times 68731.53 \times \frac{1389.6}{5280} = 279.15 \text{ miles} = h_1.$$

On the other hand, if we take the mean between the freezing and boiling points, as in Note 75, we find

68886 + $\frac{35}{15}$ of 154.47 = 69246.43 calories, representing $h_2 = 281.24$ miles.

h_1 gives $m_0 = 332,693\ m_1$; $nr = 92,871,000$ miles.
h_2 " $m_0 = 328,989\ m_1$; $nr = 92,525,000$ miles.
Note 75 $m_0 = 328,438\ m_1$; $nr = 92,472,500$ miles.

79. *Factors of Vis Viva.*

Although ordinary kinetic investigations involve considerations both of mass and velocity, the equality of action and reaction furnishes data, in many cases, for dispensing with one of the three kinetic units. For example, the equation of circular orbital revolution, $v = \sqrt{gr} = \sqrt{\frac{m}{r}}$, may be put in the form $\frac{l}{t} = \frac{\sqrt{l' l''}}{t} = \frac{\sqrt{rd}}{t}$, and we may use either of the equivalent values without vitiating our results. The meaning of the mass-factor is especially obscure in the "dimensions of electric units," but when we see that m is regarded as the product of electrostatic $m^{\frac{1}{2}}$ by electromagnetic $m^{\frac{1}{2}}$, and when we remember the importance of Ampere's theory of currents, we are reminded of the orbital equations, and if we still think of mass, it is only because we have learned to regard mass as a necessary element of force. If we look still further, and see that the product of two masses must be considered in order to secure strict accuracy in the determination of orbits, we can find no good meaning for the square root of a mass. But if electric phenomena are simply the result of oscillations in the luminiferous æther, which represent the actions and reactions of inertia and elasticity, we may reasonably look for analogous phenomena in cosmical arrangements.

80. *Cosmical Illustration.*

In the mutual actions and reactions of two masses, $v = \sqrt{gr} \propto \sqrt{\frac{m}{r}} \propto \sqrt{m}$;* $v^2 \propto m$; $\frac{m}{v} \propto v$. Let m_0, m_3, m_5 designate Sun, Earth, Jupiter; v_3, Earth's aphelion velocity; v_λ, velocity of light. Then we find

$$\frac{m_0}{v_\lambda} : \frac{m_3}{v_3} :: \frac{m_3}{v_3} : \frac{m_5}{v_\lambda}$$

For, $v_\lambda = \rho_3 \div 497.827$; $v_3 = 2\pi\rho_3 \div 1.01677 \div 31558149$; $m_0 = 1047.879\, m_5$. Substituting these values, we find: $m_5 = 316.9\, m_3$; $m_0 = 332{,}072\, m_3$; $\rho_3 = 92{,}559{,}700$ miles. Hence we see that the reaction of Earth against the forces which produce its orbital velocity, at the moment when it is least affected by Sun, is a mean proportional between the reactions of Sun and Jupiter against the velocity of light. In other words, the reaction of inertia at the centre of density (Earth) against the inertia at the centre of nebulosity (Jupiter) and at the centre of nucleation (Sun) is limited in each direction by the reactions of controlling inertia against æthereal elasticity.

Multiplying the terms of the above proportion by $v_\lambda^2 v_3$, we have

$$m_0 v_\lambda v_3 : m_3 v_\lambda^2 :: m_3 v_\lambda^2 : m_5 v_\lambda v_3.$$

The importance of the reactionary *vis viva* at the centre of condensation is thus shown, and the *vis viva* at the coördinate centres represents a mean proportional between the æthereal and the central orbital *vis viva.*

81. *Mutual Convertibility of Gravity and Magnetism.*

In the first set of electrostatic, magnetic and electrokinetic pairs, the product of the two dimensions represents an energy, or *vis viva.* Maxwell† gives the following equations:

$$[e E] = [m \Omega] = [p\, C] = \left[\frac{L^2 M}{T^2}\right].$$

The symbols denote, respectively, the products of quantity of electricity by electric potential, quantity of free magnetism by magnetic potential, electrokinetic momentum of a circuit by electric current, mass by the square of the velocity of light. *The last product is, therefore, precisely the same as* $m_3 v_\lambda^2$, *in the foregoing note,* and it furnishes further grounds for considering the primitive oscillations as luminous, rather than electric. There are as good reasons for assuming Earth's mass as the fundamental electric value of M, as for assuming the velocity of light as the corresponding value of $\frac{L}{T}$, and this assumption brings electric and thermal phenomena into simpler and more intelligible relations. The anticipations of my first paper* upon the relations of electricity and magnetism to gravitation are thus very fully and satisfactorily confirmed.

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* Because r represents the distance between the two masses and does not affect the ratio.

† Elec. and Mag., ii, 240.

82. *Chemical Units.*

The influence of electric and thermodynamic laws on chemical phenomena has long been recognized. There are two special applications of electric dimension to which I would call attention, on account of their apparent bearing upon spectral harmonies and combining equivalents.

I. The second set of electrostatic, magnetic and electrokinetic pairs gives the following equations :

$$[\mathfrak{D}\,\mathfrak{E}] = [\mathfrak{B}\,\mathfrak{H}] = [\mathfrak{C}\,\mathfrak{A}] = \left[\frac{M}{L\,T^2}\right].$$

The symbols denote, respectively, the products of electric displacement by electromotive force at a point, magnetic induction by magnetic force, intensity of electric current at a point by vector potential of electric currents, and the dimensions of energy referred to unit of volume. The last member of the equation also represents the unit of density, or atomic weight of an element, and thus opens a wide field for investigation, both in chemical and in general physics.

II. Weber found, by experiment, that the unit of electro-chemical force is to that of electromagnetic force as 106⅔ is to 1. Designating the electrochemical unit by χ and the electromagnetic unit by μ, if we suppose them to vary inversely as the squares of the reacting velocities (Note 80), we find

$$\left(\frac{m_0}{v_\lambda}\right)^2 : \left(\pi\frac{m_3}{v_3}\right)^2 :: \chi : \mu :: 106.17 : 1.$$

This differs from Weber's experimental value by less than ½ of one per cent. The π-factor designates the ratio between the time of acquiring atmospheric orbital velocity at Laplace's limit, t_a, and the time of acquiring nucleal dissociative velocity, t_n ;

$$t_a : t_n :: 1 : \pi.$$

83. *Positive and Negative.*

The oppositions of solar and Jovian photodynamic reaction, which are shown in the first proportion of No. 80, may perhaps help towards a better understanding of the difference between positive and negative electricity, or austral and boreal magnetism. While the action of æthereal oscillations must be modified by every centre of inertia, whether cosmical or molecular, the reactions of Sun and Jupiter must always greatly preponderate. In analyzing the product of the reactions into electrostatic and electromagnetic elements, $M^{\frac{1}{2}}$ may be considered as representing a mean proportional between the positive and negative masses. Reasoning from analogy, it seems probable that the electric M may represent a product, and $M^{\frac{1}{2}}$ a mean proportional, in all cases.

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* Proc. Am. Phil. Soc., ix., 355–60; April 1, 1864.

84. *Sun-Spots and Planets.*

The evidences of connection between sun-spots and planetary perturbations, the phyllotactic rhythm in periods of planetary revolution which was pointed out by Peirce and Hill,* the twelve harmonic nodes between Mercury and Sun of which I have already given evidence,† the relations of "magnetic storms" to sun-spots and to auroras, the confirmation of my first intra-Mercurial node by the British sun-spot observations,‡ as well as the various indications of a possibility that Swift's and Watson's planets may have been transient nodal aggregations, all seem deserving of consideration in studying the actions and reactions among the various forms of kinetic energy.

85. *Stellar relations of Central Density.*

In the stellar-solar paraboloid the abscissa which indicates Earth's nebula-rupturing locus, is also the locus of solar-rupture in subsidence-collision. The multiples 3, 4, 5, appear as coefficients, either of incipient subsidence or of final rupture or of both, for Earth, Mars, mean asteroid and Jupiter, their rupturing loci being, respectively, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ of their semi-axes major. The corroboration of the importance of Earth's position at the centre of density, which is furnished by Note 80 *et seq.*, is strengthened by the following multiple relations of Earth's semi-diameter, r_3:

1 $(3 \times 4 \times 5) \times r_3 \doteqdot$ Moon's semi-axis major.
2 $(3 \times 4 \times 5)^2 \times r_3 \doteqdot$ Perigee of Venus.
3 $(3 \times 4 \times 5)^3 \times r_3 \doteqdot$ Saturn's semi-axis major.
4 $4\,(3 \times 4 \times 5)^4 \times r_3 \doteqdot$ Solar modulus of light.
5 $2\pi\,(3 \times 4 \times 5)^5 \times r_3 \doteqdot$ Distance of α *Centauri.*

The deviations are of the same order of magnitude as planetary eccentricities, being as follows: 1, about $\frac{1}{2}$ of one per cent.; 2, nearly $2\frac{2}{3}$ per cent.; 3, nearly 3.4 per cent.; 4, less than $\frac{1}{24}$ of one per cent.; 5, less than $3\frac{2}{3}$ per cent. The fourth locus is equivalent to the *latus rectum* of a paraboloid in which modulus is the focal abscissa; the fifth is $\frac{\pi}{2}$ of $(3 \times 4 \times 5)$

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* Proc. Amer. Assoc., v, 2.

† Proc. Am. Phil. Soc., xix, 367.

‡ Ib., xiii, 470.

× the fourth, $\frac{\pi}{2}$ being the ratio of circular-orbital velocity to mean velocity of synchronous radial oscillation.

86. *Prime Centres of Energy.*

In considering the photodynamic reactions of Sun against the æthereal oscillations, reference has been had to Sun's centre of gravity, C_0. In my first paper* on the influence of cyclical oscillation, I anticipated the possibility of measuring fluctuations which are produced by the Central Sun. The first step in that direction was the discovery of the stellar-solar paraboloid, by means of the rupturing tendencies which would result from Herschel's theory of "subsidence," in a nebulous elastic medium. The next step was the investigation of the relations of the solar centre of gravity to the universal centre of gravity, C_u, which are involved in the cyclical equilibrium of æthereal and gravitating energies. It is perhaps possible for mathematical analysis to find some approximation to the mass and distance of the controlling centre of the immediate stellar group to which we belong, from known data. In seeking such approximation, regard should evidently be paid to the motions of the centre of gravity of the solar system, C_a. The next centre in point of importance is, perhaps, the centre of condensation in the solar system, C_3.

87. *Projection of Centres of Nucleation, Nebulosity and Condensation.*

It may be assumed that the mean centre of gravity of our system, C_α, is the same as the mean centre of gravity of its two chief masses, Sun and Jupiter. Jupiter's semi-axis major, ρ_5, is equivalent to $5.202796\rho_3$. Sun's mass, m_0, is 1047.879 times Jupiter's mass, m_5. Sun's semi-axis major, ρ_α, or mean projection from C_α, is therefore $\frac{5.202796\rho_3}{1 + 1047.879}$, or $.0049603\rho_3$. Stockwell's estimate† of the secular maximum eccentricity of Jupiter is .060874, which represents a rupturing projection, for C_α, of $\frac{1}{1 - .060874} = 1.0648r_0$. The equation, $.0049603\rho_3 = 1.0648r_0$, gives $\rho_3 = 214.667r_0$; $n = 214.667$. This is about $\frac{1}{10}$ of one per cent. greater than the estimate of the British Nautical Almanac, and about $\frac{1}{10}$ of one per cent. less than that of the American Nautical Almanac.

88. *Orbital and Nascent Energies at the Centre of Condensation.*

The evidences of photodynamic projection, which were presented in Note 80, together with other evidences of the importance of Earth's mass and semi-diameter, as secondary thermal and electric units, enable us to connect orbital velocity, v_α, with nascent or dissociative velocity, v_β, at the centre of condensation, Earth. For, since distance or *vis viva* of projection against uniform resistance varies as the square of the time of communicating or overcoming the velocity of projection, we find the proportion: Sun's projectile locus or semi-axis major, ρ_α, *is to* Earth's projectile locus or semi-axis major, ρ_β *as* the square of the time of communicating orbital velocity, t_α^2, *is to* the square of the time of communicating nascent or dissociative velocity, t_β^2. Assuming the values, $\rho_\alpha = 1.0648 r_0$, $\rho_\beta = 214.667 r_0$, in accordance with the foregoing note, and remembering that t_β is always the time of a half rotation, we have

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* Proc. Am. Phil. Soc., ix, 288.

† Smithsonian Contributions, 232, p. 38.

$$1.0648 : 214.667 :: t_\alpha^2 : 43082^2$$

$$t_\alpha = 3034.25 \text{ sec.}$$

$$v_\alpha = 3034.25 \times 32.088 \div 5280 = 18.44 \text{ miles.}$$

$$\rho_3 = 18.44 \times 31558149 \div 2\pi = 92,617,300 \text{ miles.}$$

89. *Double Systems of Photodynamic Oscillation.*

In 1864, I suggested* "that one of the most probable results of the rotation of the earth with its atmosphere, in an æthereal medium, would be the production of two systems of oscillations, moving with the rapidity of light, one in the line of the Earth's orbit, and the other in the line of its radius vector, and that those systems would be constantly so related that while one tended to retard, the other would tend to accelerate the Earth's motion." In 1866, I showed† that "the velocity of light is nearly the same as the velocity which would be acquired in one year by a falling body, under the influence of an accelerating force equivalent to the force of gravitation at the Earth's surface." Although I regarded this approximation, at first, as "perhaps merely a curious accidental coincidence," it became so prolific in suggestive leadings that I was able to show its dependence upon simple dynamic laws of æthereal condensation. From the electrostatic analogies of condensation and the electrokinetic analogies of orbital motion, we may infer the probable applicability of the law of energy and stress of radiation, "that at every point of the wave the intrinsic energy of the medium is half electrostatic and half electrokinetic."‡ At the locus of primitive coincidence between orbital and rotary velocity, this division of energy would lengthen the time of acquiring the solar nascent dissociative velocity of light to a complete rotation of the centre of condensation, the time for the nucleus being a half-rotation. Dividing one year by the square root of the ratio of solar photodynamic projection (Note 88), we find 31558149 seconds $\div \sqrt{1.0648} = 3058254$ sec.; $v_\lambda = 3058254 g = 185,858$ miles; $\rho_3 = 497.827 v_\lambda = 92,525,300$ miles. This differs by less than one-tenth of one per cent. from the value given in the foregoing note.

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* Proc. Am. Phil. Soc., ix, 408.

† Ib., x, 269.

‡ Maxwell, ii, 391.

90. *Formulas for Conversion of Energy.*

All forms of energy, mechanical, thermal, photic, electric, magnetic or chemical, are derived from and can be compared with the maximum energy, $\frac{M_0 L_0^2}{T_0^2}$.

α. All astronomical, barometric, or other mechanical estimates of solar mass and distance, involve the proportionality, $g \propto \frac{m}{l^2}$.

β. The arbitrary units of thermodynamics are based upon the work done against the centripetal accelerations of terrestrial gravity. The unit of acceleration at unit of distance, or the sum of accelerations in unit of time, is proportioned to mass. Therefore, if we designate Earth's mass by m_3, we have $M_0 : m_3 :: V_0 : u_3$.

V_0, or the velocity of light is, as we have seen, the velocity acquired during the cyclical actions and reactions of solar condensation and æthereal elasticity at Sun's surface; $u_3 = .56558$ miles, is the velocity acquired during the cyclical actions and reactions of water-congelation and evaporation at Earth's surface. Dividing by $\sqrt{180}$ for the Fahrenheit scale, or by $\sqrt{100}$ for the Centigrade scale, we have the arbitrary units of velocity, .042156 miles for 1° F., or .056558 miles for 1° C. The equation $v = \sqrt{2gh}$ gives $h = 772$ ft. for 1° F. or 1389.6 ft. = 424 meters for 1° C. Combining these heights with the arbitrary units of mass we have J = 772 ft. lb. for the English thermal unit, and C = 424 kilogrammetres for the calorie or French thermal unit.

γ. Thermal, mechanical and photodynamic energies may be compared with energies of chemical combination through the ratio

$$M_0 : m_3 :: V_0 : u_3 :: h_0 : h_2.$$

h_0 is Earth's mean distance from Sun, h_2 is $\frac{1}{4}$ of $\frac{5}{9}$ of the height to which water-vapor would be thrown, against the retardation of gravity, by the combining energy of H_2O (Note 16, etc.).

δ. Cosmical, electric and photodynamic energies may be compared by means of the ratio

$$M_0 V_0 v_3 : m_3 V_0^2 :: m_3 V_0^2 : m_5 V_0 v_3.$$

Sun, M_0, is at the centre of nucleation in the solar system; Earth, m_3, at the centre of condensation; Jupiter, m_5, at the nebular centre; v_3, is Earth's aphelion or "nascent" orbital velocity.

ε. The electrostatic, magnetic and electrokinetic units of energy can all be derived from the above expression for Earth's photodynamic energy, $m_3 V_0^2 = \frac{m_3 L_0^2}{T_0^2}$, through the equations

$$[e\, E] = [m\, \Omega] = [p\, C] = \frac{M L_0^2}{T_0^2}$$

The bracketed symbols represent, respectively, quantity of electricity; line integral of electromotive force, or electric potential; quantity of free magnetism, or strength of a pole; magnetic potential; electrokinetic momentum of a circuit; electric current.

ζ. Atomic energy, or energy of unit volume, can be compared with Earth's photodynamic energy of unit volume, $\frac{m_3 L_0^2}{r_3^3 T_0^2} = \frac{m_3}{L_1 T_0^2}$, and with corresponding electric energies, through the equations

$$[\mathfrak{D}\,\mathfrak{E}] = [\mathfrak{B}\,\mathfrak{H}] = [\mathfrak{C}\,\mathfrak{U}] = \frac{M}{L T^2}$$

The bracketed symbols represent, respectively, electric displacement (measured by surface-density); electromotive force at a point; magnetic induction; magnetic force; current electric intensity at a point; vector potential of electric current.

η. Electrochemical and electromagnetic energies may be compared with thermal, photodynamic and other energies, through the proportion

$$\chi : \mu :: M_0 t_a^2 \times M_0 v_3^2 : m_3 t_n^2 \times m_3 V_0^2$$

I designate Weber's units of electrochemical and electromagnetic force by χ and μ, respectively; t_a is the time of acquiring orbital velocity, or incipient associative energy, at Laplace's limit of equal velocities of rotation and revolution; t_n is the time of acquiring nucleal nascent or dissociative velocity.

$$t_a : t_n :: 1 : \pi$$

θ. Total magnetic force, φ_0, can be compared with the reactions of terrestrial magnetic force, φ_3, by the proportion

$$M_0^2 : \pi^4 m_3^2 :: \varphi_0 : \varphi_3$$

The reactions of orbital tendency are $t_a M_0^{\frac{1}{2}}$, $t_n m_3^{\frac{1}{2}}$, respectively; centripetal acceleration varying as the fourth power of orbital velocity, we have the ratio, $M_0^2 : \pi^4 m_3^2$.

(Contributions from the University of Pennsylvania, No. 16.)

On Alaskaite, a new member from the series of Bismuth Sulphosalts. By Professor George A. König.

(Read before the American Philosophical Society, June 17, 1881.)

Mode of occurrence. The high mountain mass of south-western Colorado culminates in Mt. Sneffels at an altitude of 14,156 feet. But this great height does not impress itself upon the observer since the deepest valleys are still 7000 feet above the sea. The geological structure of these mountains is very simple as a whole; but the energetic action of the forces of erosion, has produced a very rich and picturesquely carved topography, there being no table mountains, but only steep peaks and sharply indented crests. Mineral veins in clearly defined outcrops intersect these moun-